A model for phase transitions:

$$E = -\underbrace{\mu_0 H}_{\equiv h} \sum_i S_i - J \sum_{i,j}{}' \underbrace{S_i S_j}_{\substack{=1 \text{ if aligned} \\ -1 \text{ if not}}}$$

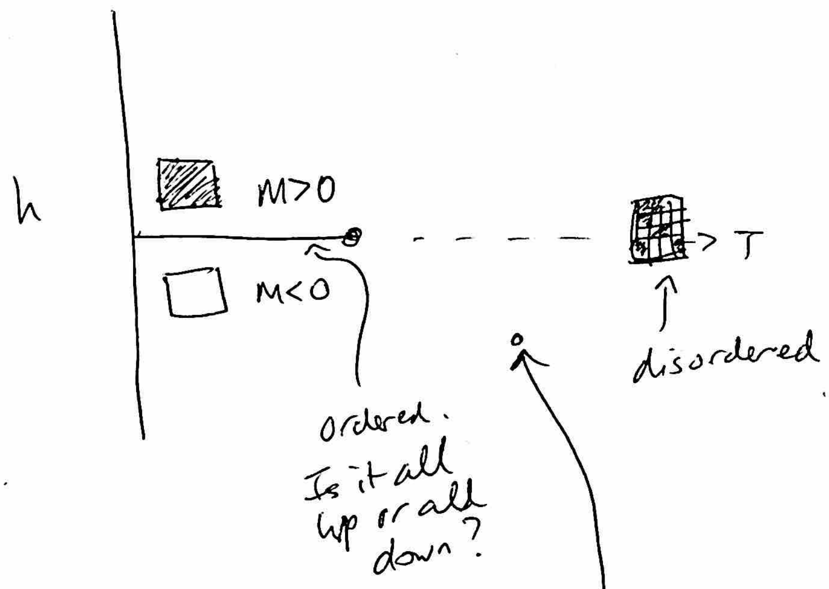
(note up-down symmetry if $h=0$)

Canonical partition function

$$Q = \sum_{S_1=\pm 1} \sum_{S_2=\pm 1} \cdots \sum_{S_N=\pm 1} e^{\beta h \sum_i S_i + \beta J \sum'_{i,j} S_i S_j}$$

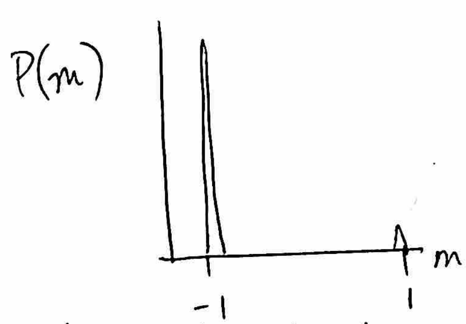
$M = \sum_i S_i$ net magnetization

$m = \frac{1}{N} \sum_i S_i$ net magnetization per site



Yikes, the problem set used M for the # of cells

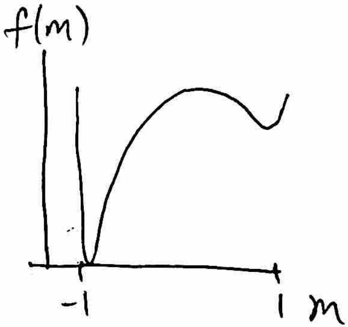
Pick one point in phase space and collect the probability distribution for m.



$$P(m) \sim e^{-N \overset{\beta}{\underbrace{f(m)}}}$$

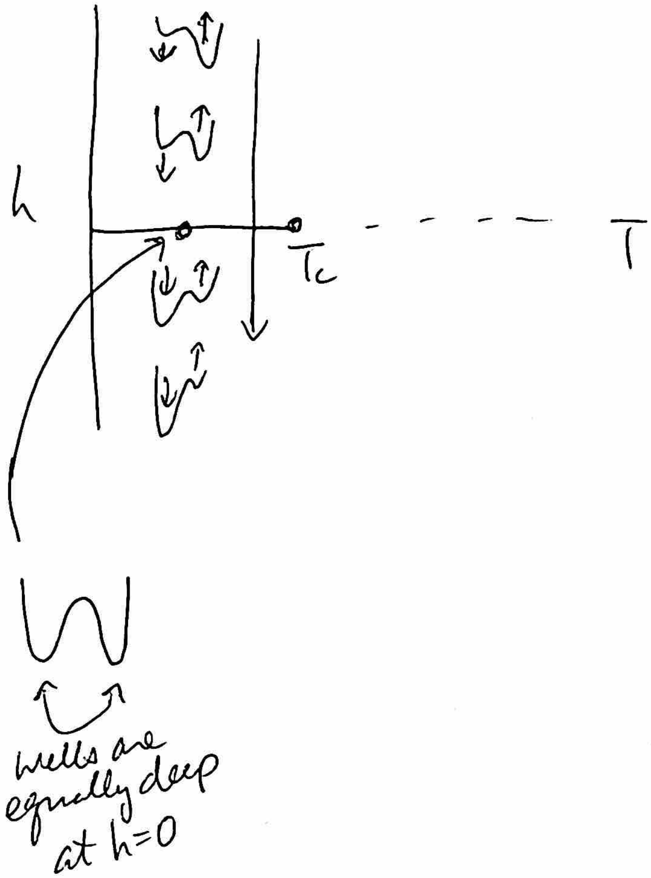
N: # of spins

$f(m)$: free energy of magnetization (per spin)

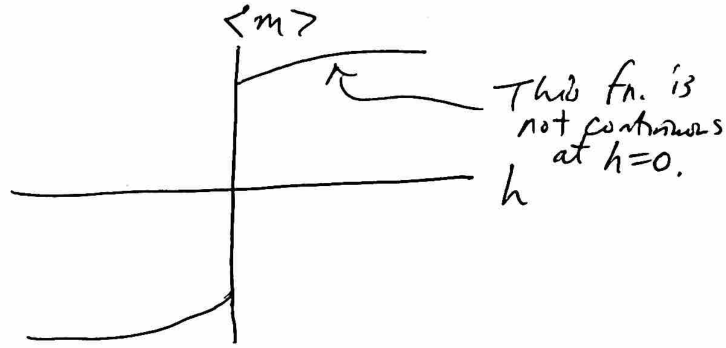


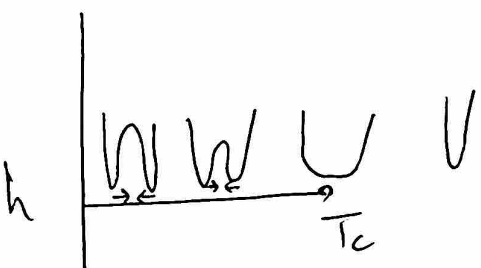
You will learn how to use Monte Carlo sampling to collect this distribution

There are different types of phase transitions...



Whether the left or the right well is lower changes when $h>0$ vs. $h<0$, so which minimum we see jumps discontinuously, resulting in a first order transition





The location of the minima shift inward until they merge together at $T=T_c$ to give a second order transition...