

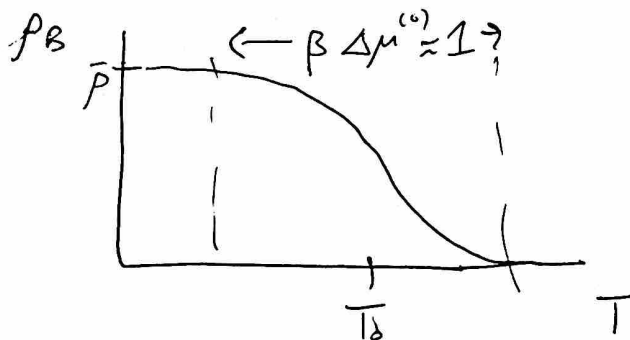
Crossovers and Phase Transitions

$$B \rightleftharpoons C$$

$$\frac{P_C}{P_B} = e^{-\beta \Delta \mu^{(0)}}$$

$$\Delta \mu^{(0)} = \frac{\Delta h^{(0)}}{T} - \Delta s^{(0)}$$

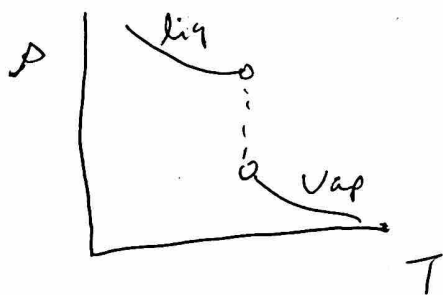
Both positive, ind. of T, large



$$P_B = \bar{P} - \frac{\bar{P}}{1 + e^{\beta \Delta \mu^{(0)}}}$$

I put some numbers in the last lecture notes, but suffice it to say the crossover is quite broad. For a very exothermic reaction ($\Delta h^{(0)} = 600 k_B T_0$), $\Delta T \approx$ a few degrees C.

By contrast, for liquid-vapor transition at fixed ρ



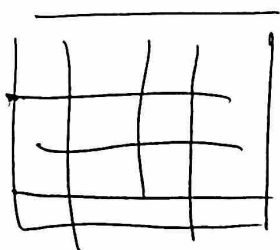
$$\Delta T \approx 0.$$

"Non-analytic" behavior defines the phase transition. What's so special about this?

$$P_{liq} = \frac{Q_{liq}}{Q} \quad \text{where} \quad Q_{liq} = \int_{liq} dr^N \frac{e^{-\beta U(r^N)}}{\int_{smooth \text{ fn. at finite } T}}$$

Q_{liq} & Q are sums of smooth functions... they can't be non-analytic!
What gives? Strict phase transitions are only possible when $N \rightarrow \infty$

We can always divide a macroscopic system into many (M) uncorrelated subsystems...



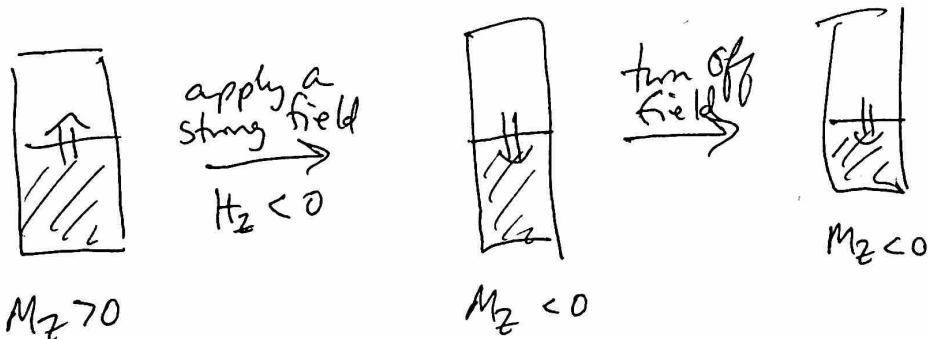
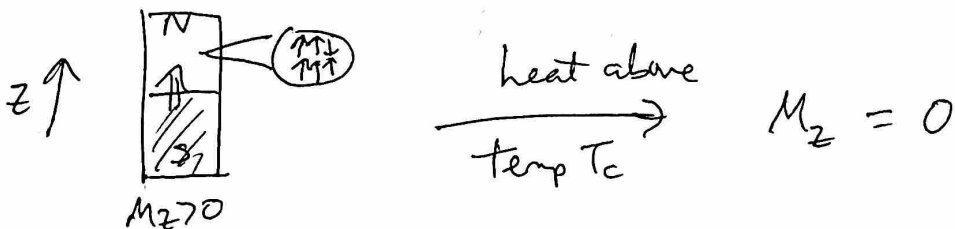
Let g_{cell} = partition fn. for all flucts. in given cell
 = smooth fn.

$$Q = g_{\text{cell}}^M \Rightarrow -\beta A = \ln Q = M \ln g_{\text{cell}}$$

So A/M is a smooth fn too \Rightarrow no phase transitions!

Evidently phase transitions involve fluctuations which are correlated over macroscopic distances!

Very similar to liq-vapor transition is the phase transition in magnets.



If $H_2 = 0$, the Hamiltonian has up-down symmetry

$\begin{bmatrix} \uparrow \uparrow \uparrow \\ \downarrow \uparrow \uparrow \end{bmatrix}$ has the same energy as $\begin{bmatrix} \downarrow \downarrow \downarrow \\ \uparrow \downarrow \downarrow \end{bmatrix}$, so they have the same eq. weight, right?

Below T_c , the equilibrium state does not ~~have~~ possess this symmetry!

* Spontaneous symmetry breaking is central to statistical mechanics of phase transitions

We then discussed the Ising model

$$E = -\frac{\mu H}{h} \sum_i S_i - J \left(\sum_{\langle ij \rangle} S_i S_j \right)$$

Prime denotes we only include nearest neighbor spins

Each pair of neighbors is included one time, but it may be simpler to execute in a program if one sums over both $S_i S_j$ and $S_j S_i$ with $J/2$ out front.

We worked through an example of computing the energy and we talked about how the coupling term J complicates the calculation of the canonical partition function $Q(\beta)$.