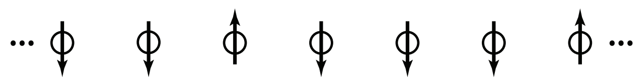
1. **Independent spins in a magnetic field. [65 pts.]** Consider a lattice of N noninteracting spin-1/2 particles in a magnetic field. (Because each particle is fixed at a particular lattice site, different particles can be distinguished from one another.)

   Each particle has two possible eigenstates, with energies $\epsilon/2$ ("spin up") and $-\epsilon/2$ ("spin down"). An example configuration of the system is shown below.

   The total energy of particles shown above is $5(-\epsilon/2) + 2(\epsilon/2) = -3\epsilon/2$. This problem will consider the entropy of the spin system and the equilibrium that is attained when the spins can exchange energy with a temperature bath.

2. **Pulling on a Polymer. [35 pts.]** Consider a classical polymer in three dimensions, which consists of $N+1$ monomers connected together in a linear chain by N harmonic springs. We use $\vec{R}_i$ to denote the position of the i^{th} monomer. The energy of the polymer has kinetic and potential energy components, which depend on the positions and velocities of every monomer.

$$E(\vec{R}_0, \ldots, \vec{R}_N; \dot{\vec{R}}_0, \ldots, \dot{\vec{R}}_N) = \frac{k}{2} \sum_{i=1}^{N} \left| \vec{R}_i - \vec{R}_{i-1} \right|^2 + \frac{m}{2} \sum_{i=0}^{N} \left| \dot{\vec{R}}_i \right|^2,$$

   where m is the mass of a monomer, $\dot{\vec{R}}_i$ is the velocity of monomer i, and the strength of the spring connecting polymers is k with units of energy per length squared. The polymer is immersed in a solution which has temperature T, so the energy of the polymer can fluctuate. The first part of the problem concerns this free polymer where "free" indicates that we are not applying extra forces to hold the endpoints of the polymer fixed in space.

   Later in the problem we will imagine fixing monomer 0 at the origin by, for example, attaching it to a glass bead and holding that bead at the origin with a pipette. Monomer N will be fixed at $L\hat{x}$, which could be physically realized by attaching that monomer to a different glass bead and moving the bead with an optical trap. You will not need to know anything about how such an optical trap functions, only that it is possible to move the two endpoints of the polymer relative to each other so that they are separated by a distance L.