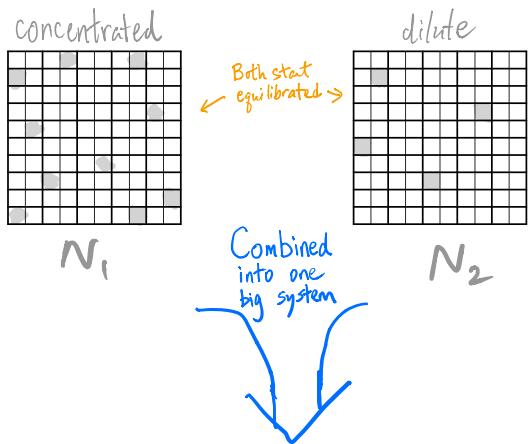


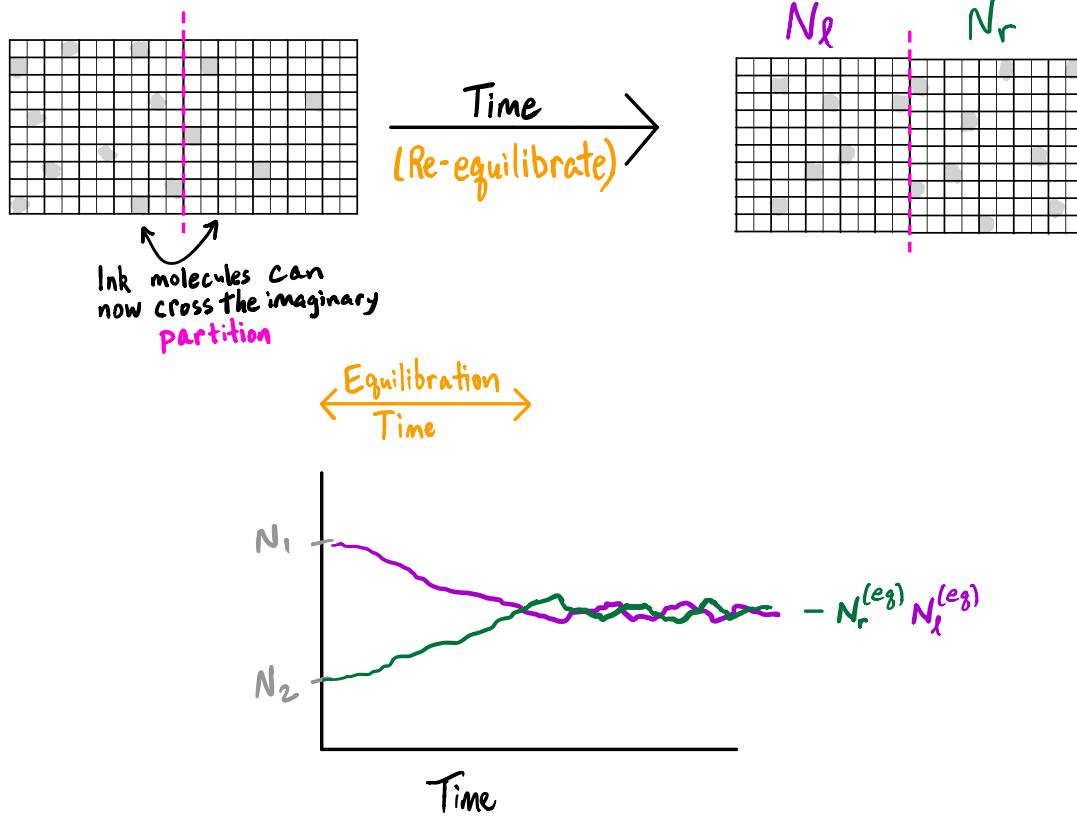
## Lecture 8

Recall from last lecture...



1. Why do particles repartition?

2. What is the condition that determines the equilibrium partitioning?



Equilibration will induce changes (repartitionings) until the new combined system is in its most likely partitioning - the partitioning with the most microstates

The most likely partitioning will maximize

$$S(\Delta N) = S_l(N_l^{(eq)} + \Delta N) + S_r(N_r^{(eq)} - \Delta N)$$

[or equivalently minimize  $-S(\Delta N)$ ]

$$\Rightarrow 0 = \left. \frac{dS(\Delta N)}{d\Delta N} \right|_{\Delta N = \Delta N^{(eq)}}$$

$$= \left. \left( \frac{\partial S}{\partial N} \right)_{E,V} \right|_{\Delta N = \Delta N^{(eq)}} - \left. \left( \frac{\partial S}{\partial N} \right)_{E,V} \right|_{\Delta N = \Delta N^{(eq)}}$$

Remember  $N_l = N_l^{(eq)} + \Delta N$   
 $N_r = N_r^{(eq)} - \Delta N$  and  $\Delta N^{(eq)} = 0$

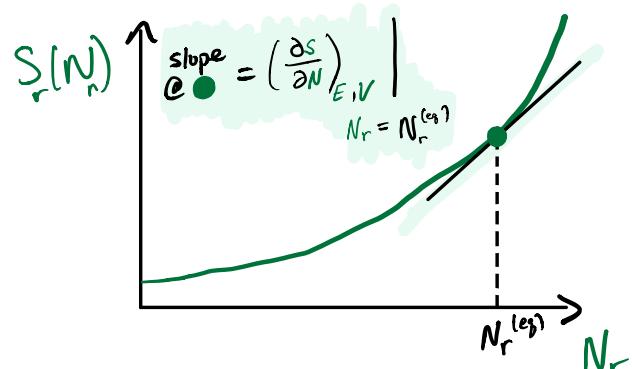
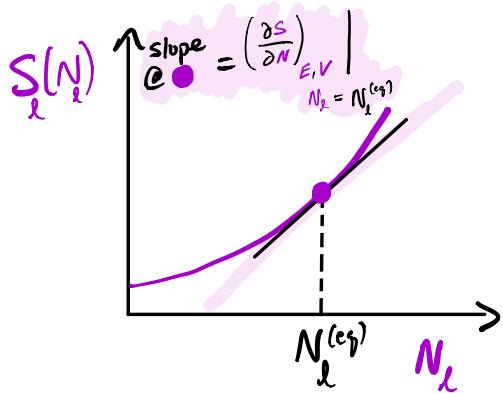
$$= \left. \left( \frac{\partial S}{\partial N} \right)_{E,V} \right|_{N_l = N_l^{(eq)}} - \left. \left( \frac{\partial S}{\partial N} \right)_{E,V} \right|_{N_r = N_r^{(eq)}}$$

So at equilibrium it must be the case that

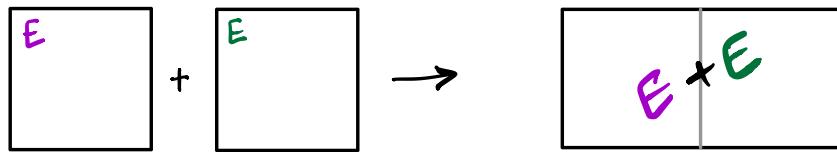
$$\left(\frac{\partial S}{\partial N}\right)_{E,V} \mid_{N_L = N_L^{(eq)}} = \left(\frac{\partial S}{\partial N}\right)_{E,V} \mid_{N_r = N_r^{(eq)}}$$

Particles repartition until the slopes of entropy with respect to changing particle number are equal.

An illustration...



Repeat with energy partitioning...

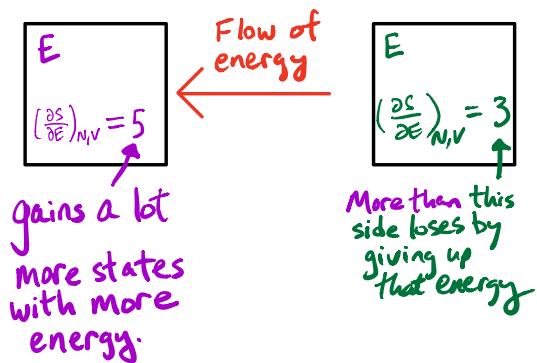


Energy flows until  $(\frac{\partial S}{\partial E})_{N,V} = (\frac{\partial S}{\partial E})_{N,V}$

Let's think about our colloquial meaning of "temperature"

Energy flows until  $T = T$

So...  $T = \left(\frac{\partial S}{\partial E}\right)_{N,V}$  ? Not quite



Flow of energy goes from low  $(\frac{\partial S}{\partial E})_{N,V}$  to high  $(\frac{\partial S}{\partial E})_{N,V}$ .

So...  $T = -\left(\frac{\partial S}{\partial E}\right)_{N,V}$  ? I don't love those negative temperatures

Instead,

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{N,V}$$

Let's view our thermodynamic expression

$$dS = \frac{1}{T} dE + \frac{P}{T} dV - \frac{\mu}{T} dN$$

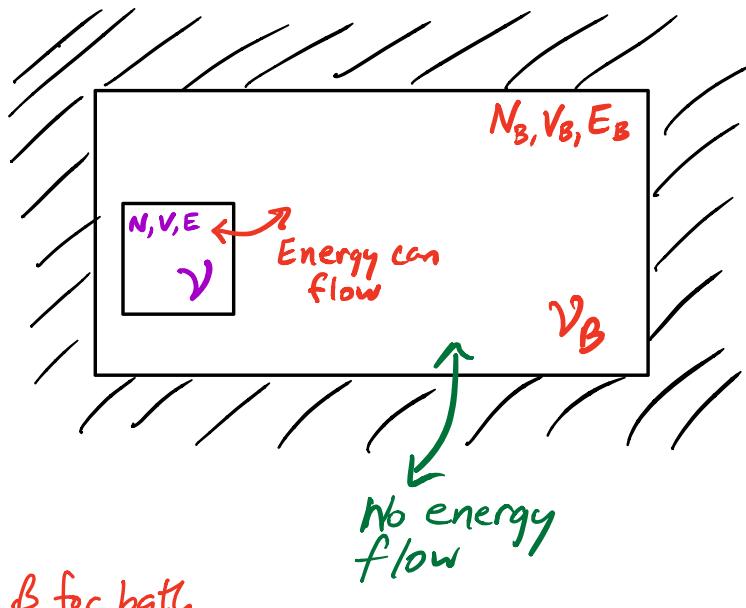
in terms of statistical mechanics (counting that is)

We are thinking of  $S(E, V, N)$  which counts the # of microstates (log scale) as a function of the three constraints:  $E, V, N$

But some day we may choose to change/relax one of the constraints. By how much would  $S$  change in response?

$$\left(\frac{\partial S}{\partial E}\right)_{V,N}; \quad \left(\frac{\partial S}{\partial V}\right)_{E,N}; \quad \left(\frac{\partial S}{\partial N}\right)_{E,V}$$

How do you relax those constraints?

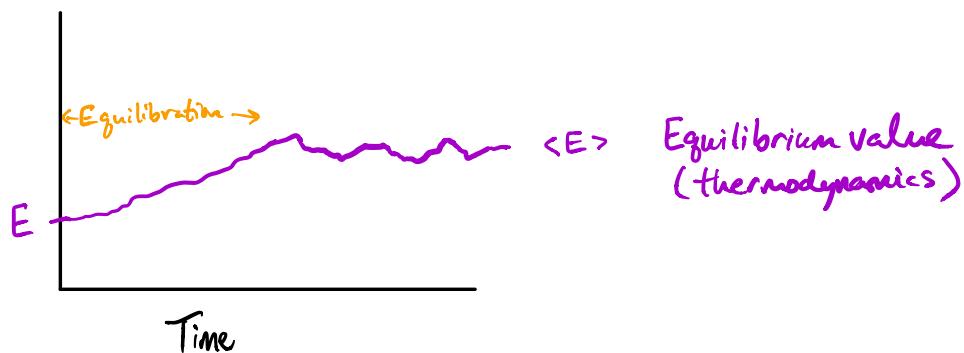


$\beta$  for bath



Now we have a fluctuating  $E$  but a fixed  $\beta$   
Why a fixed value of  $\beta$ ?

$$[\beta = \frac{1}{k_B T} = \frac{1}{k_B} (\frac{\partial S}{\partial E})_{N,V} \text{ set by bath}]$$



Some questions:

1. What is the expected equilibrium energy and how does it depend on the bath temperature?
2. What sets the scale of the fluctuations?

We have already seen that

$$P(v) \propto e^{-\beta E(v)} \quad \begin{matrix} \text{(Boltzmann} \\ \text{factor)} \end{matrix}$$

$\boxed{N, V, \beta}$   
 $v$

- Why is this merely proportional and not equal?

We need to normalize the distribution.

- How do I normalize?

$$1 = \sum_v P(v) \quad \text{and} \quad P(v) = \frac{e^{-\beta E(v)}}{Z}$$

Normalization  
Constant

$$\Rightarrow 1 = \sum_{\nu} \frac{e^{-\beta E(\nu)}}{Z} \Rightarrow$$

"Canonical Distribution"

$$P(\nu) = \frac{e^{-\beta E(\nu)}}{Z}$$

Where  $Z = \sum_{\nu} e^{-\beta E(\nu)}$  (Canonical partition function)

Let's compute the typical value of energy...

$$\langle E \rangle = \sum_{\nu} E(\nu) P(\nu) = \sum_{\nu} E(\nu) \frac{e^{-\beta E(\nu)}}{Z}$$

$$= \frac{1}{Z} \sum_{\nu} E(\nu) e^{-\beta E(\nu)}$$

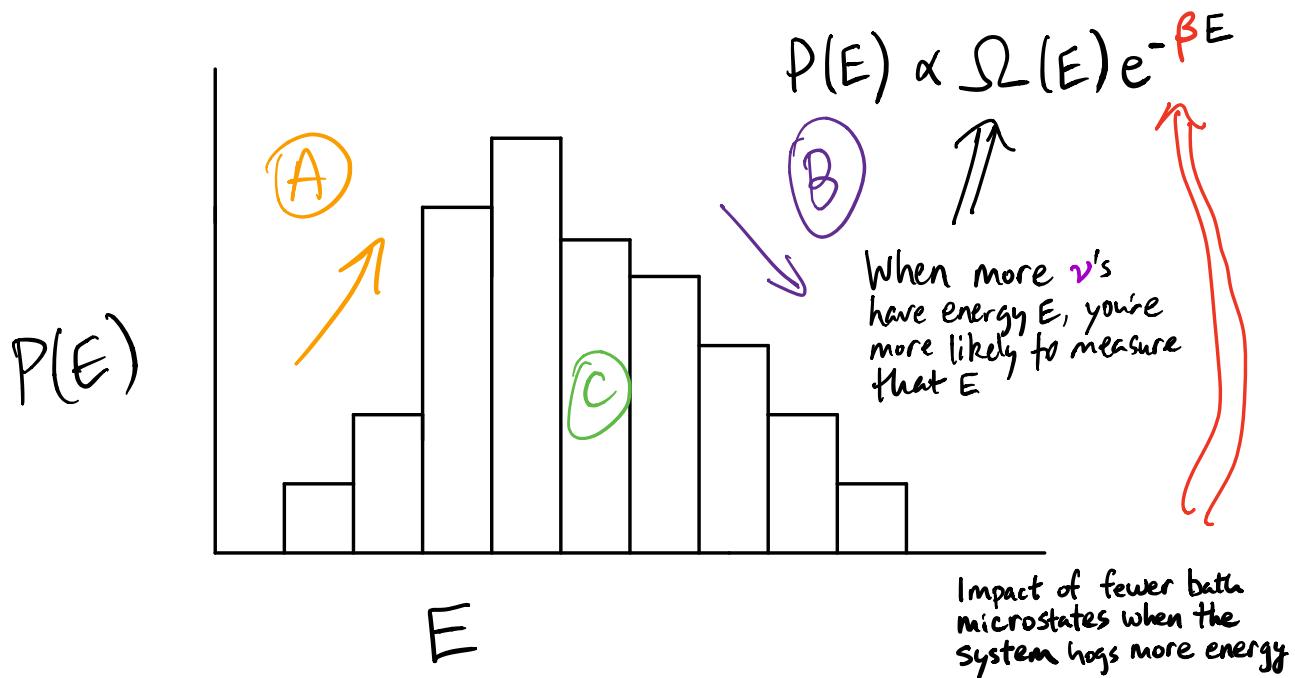
$$= \frac{1}{Z} \sum_E \sum_{\substack{\nu \text{ with} \\ \text{Energy } E}} E(\nu) e^{-\beta E(\nu)}$$

$$= \frac{1}{Z} \sum_E E e^{-\beta E} \sum_{\substack{\nu \text{ with} \\ \text{Energy } E}} 1$$

$\Omega(E) = \sum_{\substack{v \text{ with} \\ \text{Energy } E}} 1$  is called the density of states. It counts how many  $v$ 's have energy  $E$ .  
 (Equivalently degeneracy)

$$= \frac{1}{Z} \sum_E \Omega(E) E e^{-\beta E}$$

$$= \sum_E P(E) E, \text{ where } P(E) \propto \Omega(E) e^{-\beta E}$$



(A) There are more ways to configure a system as more energy is added, increasing the system multiplicity  $\Omega(E)$ .

(B) As you put more energy in the system, you're requiring the bath to supply more, thereby lowering the bath multiplicity  $\Omega_B(E_T - E)$ .

(C) Within any one bar of fixed  $E$ , all microstates are equally likely.

(That uniform distribution over microstates)  
is called "microcanonical".