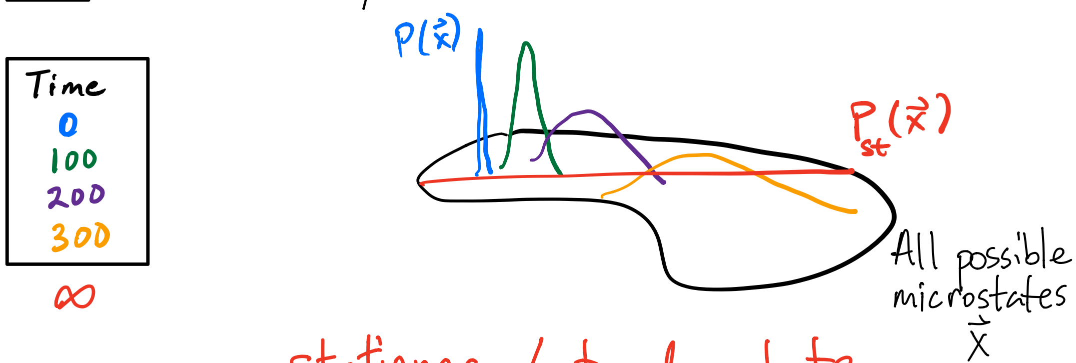
Lecture 3

Recall from last lecture...

Two major types of simplifications:

$\boxed{A}$ Ditch the dynamics $\Rightarrow$ Distributions over state space



stationary / steady state

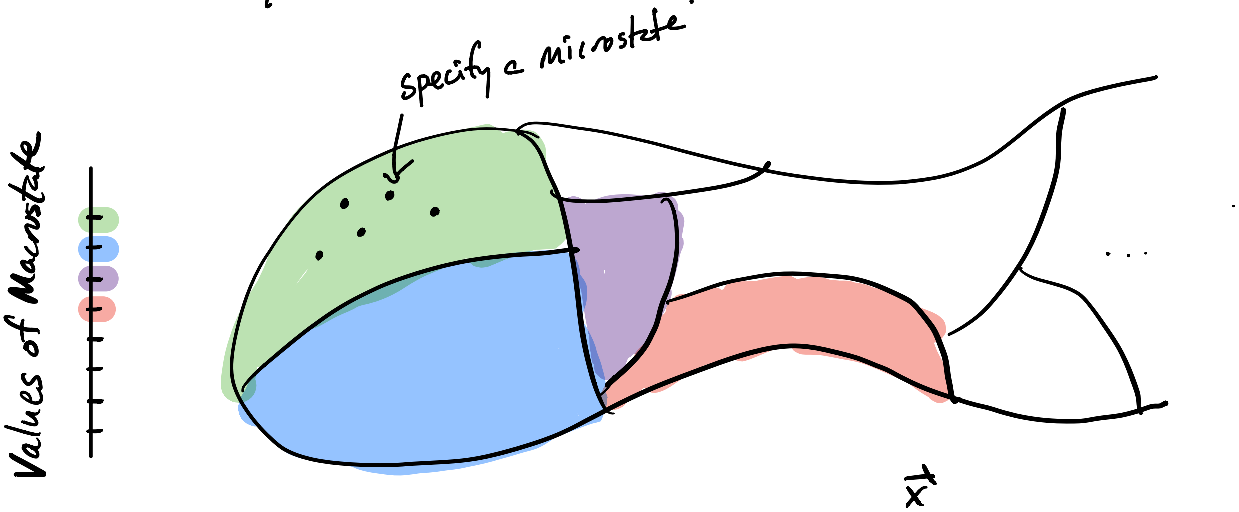
PRINCIPLE OF EQUAL A PRIORI WEIGHTS

$$P(\vec{x}) \propto \begin{cases} 1 & , \vec{x} \text{ satisfies conservation laws } (E = E_{init}) \\ 0 & , \text{otherwise} \end{cases}$$

A uniform distribution over the states

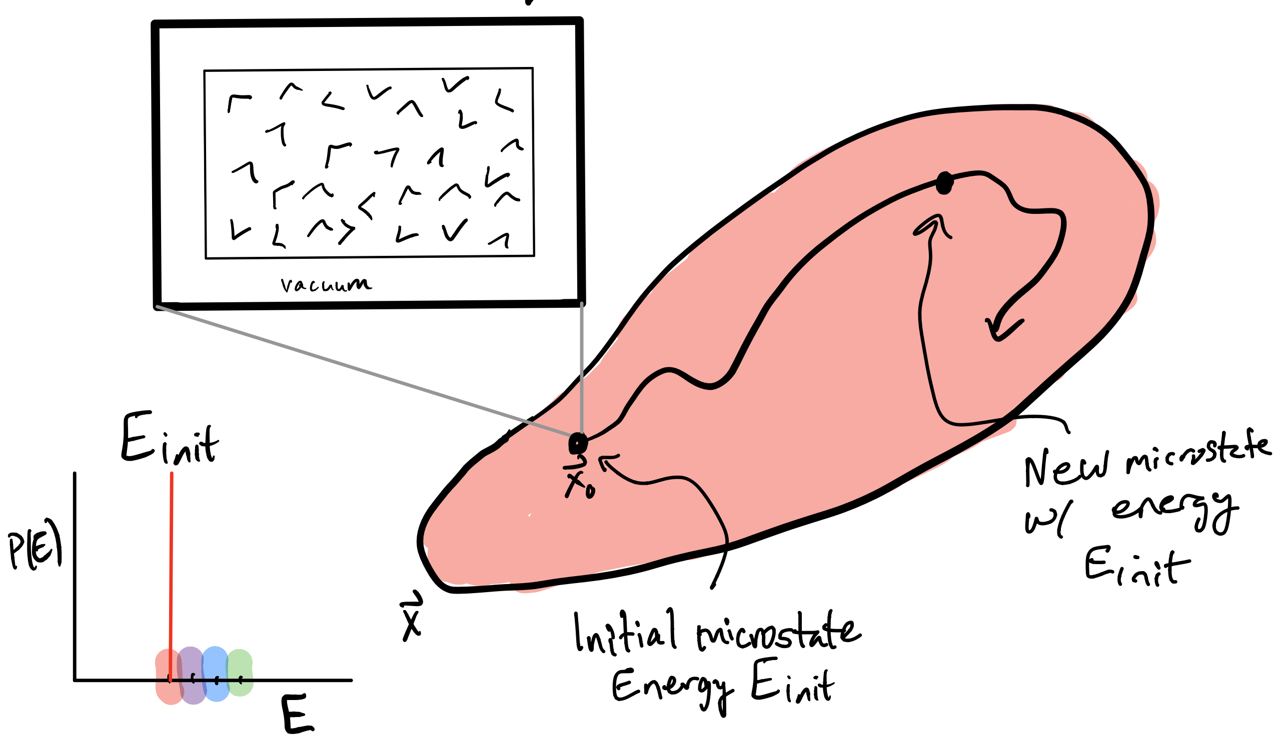
(But only those states which are possible)

B Don't try to measure everything! (you can't anyways)

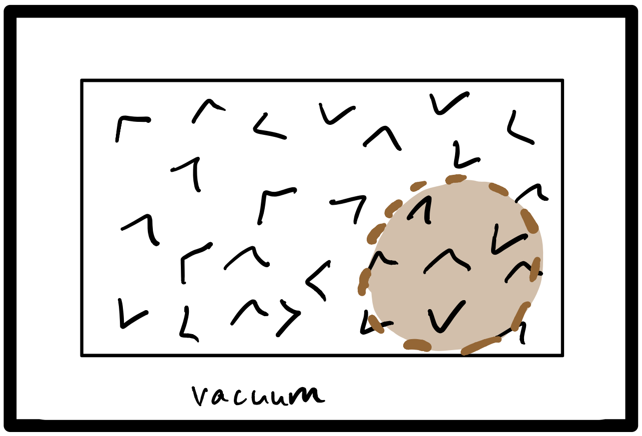


(a many-to-one mapping if you like)

Closed, isolated system with N H_2O molecules

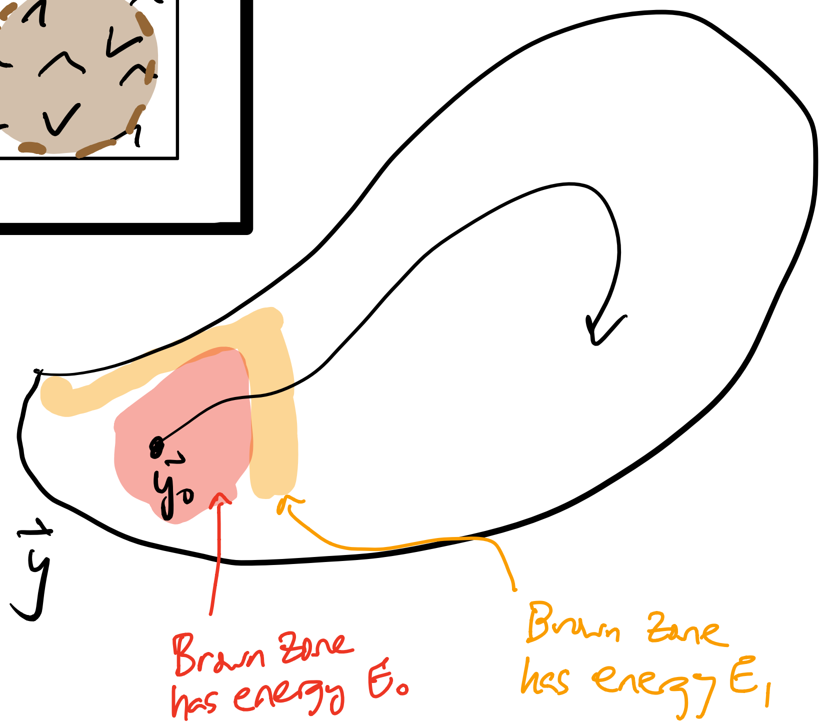


What is P(E) for the subsystem?



What is $\vec{y}$?

A vector w/ all degrees of freedom for particles inside the brown zone



Conservation laws (# of particles + energy) no longer restrict the evolution of $\vec{y}$ because particles can leave/enter the brown region.

We started with a uniform distribution as a physically motivated guess/postulate

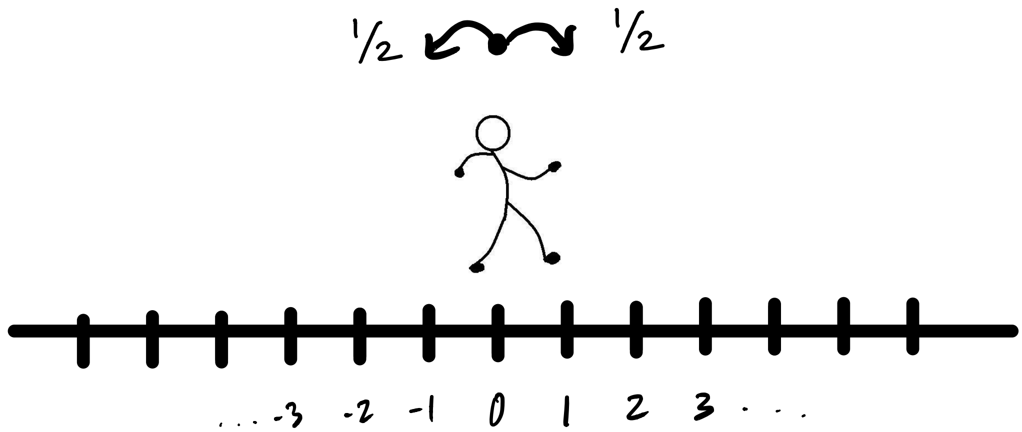
Nevertheless, we've landed on a distribution of energies which is not uniform!

Apparently marginalization due to incomplete measurements can have a very interesting effect!

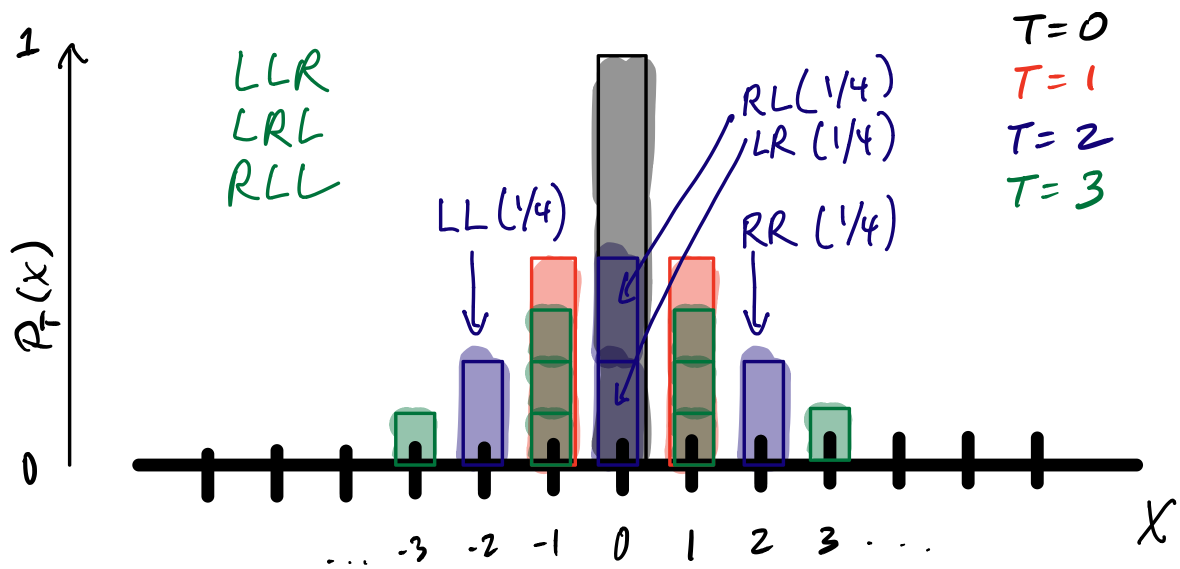
Let's dig more into an example of equal a priori weights yielding a non-uniform distribution.

"A random walker in 1D"

(A model that explains diffusion)



After T steps, what is the probability of the walker being at position X, $P_T(X)$?



Notice that (T=2)

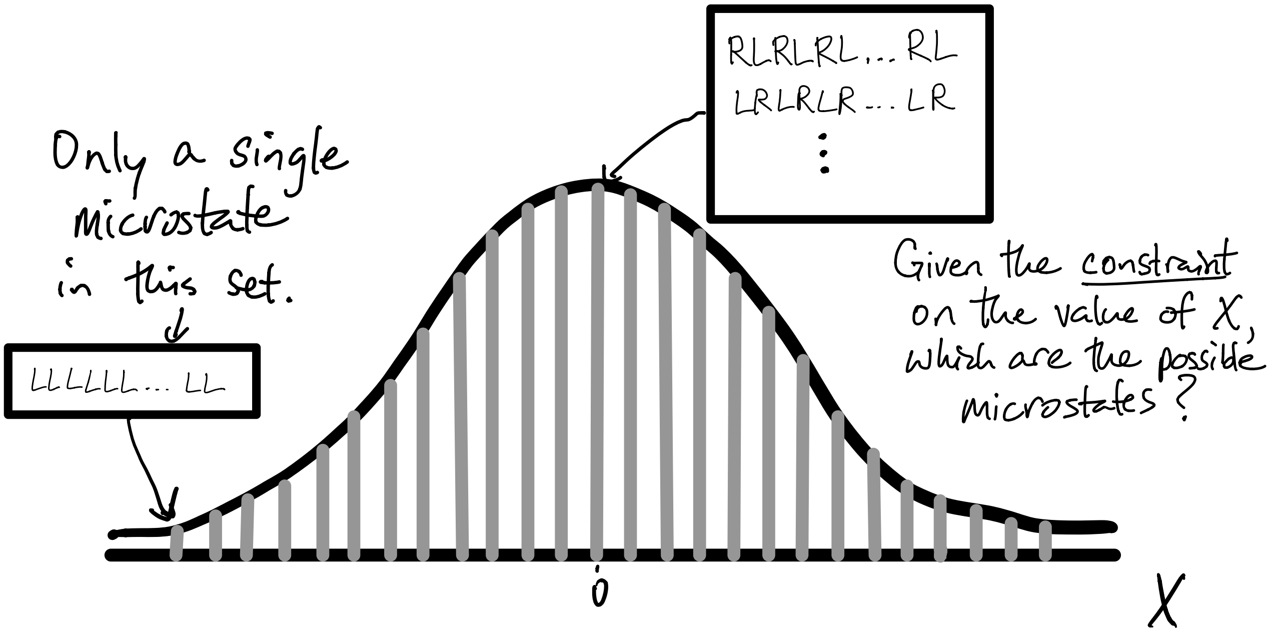
$$P_2(LL) = P_2(RL) = P_2(LR) = P_2(RR) = 1/4$$

but

$$P_2(X=2) \neq P_2(X=0)$$

Just counting. Nothing fancy!

After a LONG time...



$$\boxed{P_T(x) = \frac{{}_T C_{\frac{x+T}{2}}}{2^T}}$$

← Combinatorial Factor: How many microstates satisfy the constraints

Fraction of microstates which satisfy the constraint X at time T

Number of possible microstates

$${}_N C_M = \frac{N!}{M!\,(N-M)!}$$

Hmm... Somehow connected to entropy...?

Question: After $T=50$ steps...

What is more likely,

RRRR... RR or LRLR... LR?

Microstates all have the same probability

What is more likely,

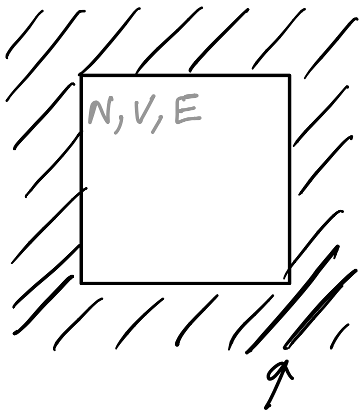
50 R's and 0 L's or $\boxed{\text{25 of each}}$?

Macrostates don't.

These are two very different questions, but often in seeing a sequence like LRLRLR..., we mistakenly view it as the set of all 25/25 mixtures.

We don't make the same mistake with RRRR... RR because it is the only element in the set.

Let's jump back to the closed, isolated system.



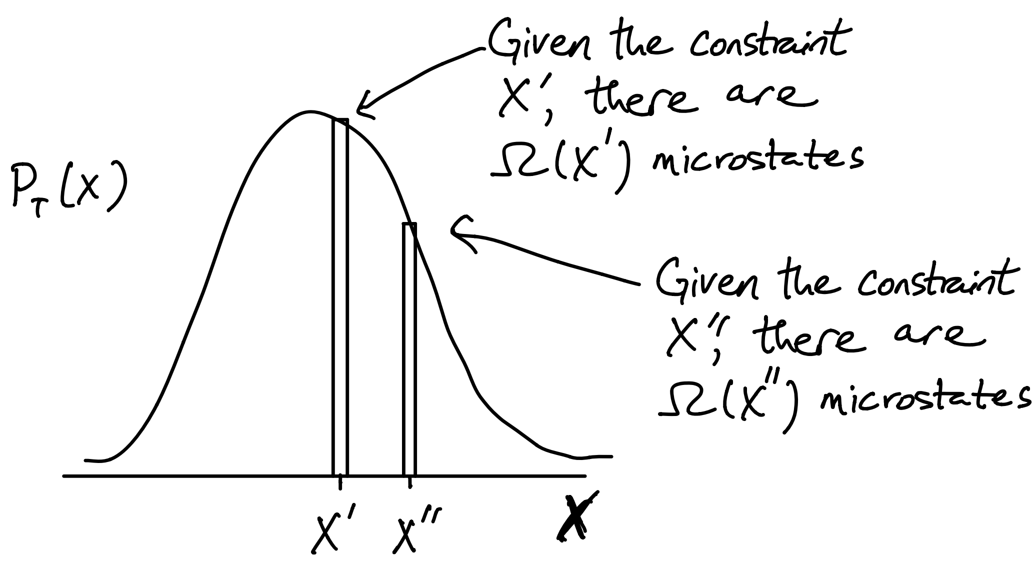
Label of the fixed constraints

These lines indicate isolation from the surroundings — no particles, energy, volume can be exchanged

$\Omega(N, V, E) \equiv$ Total # of microscopic states consistent w/ constraints N, V, E

definition

We've seen something like this!



Now I want the conservation laws to be the things providing constraints.

What is the probability of any one microstate ν?

They're all equally likely (given the satisfy the constraints), so...

$$P(\nu) = \begin{cases} \dfrac{1}{\Omega(N,V,E)} & , \text{ if } E_\nu = E,\ N_\nu = N,\ V_\nu = V \\ 0 & , \text{ otherwise} \end{cases}$$

where $\Omega(N,V,E) \equiv$ Total # of microscopic states consistent w/ constraints N, V, E

(a normalization factor)