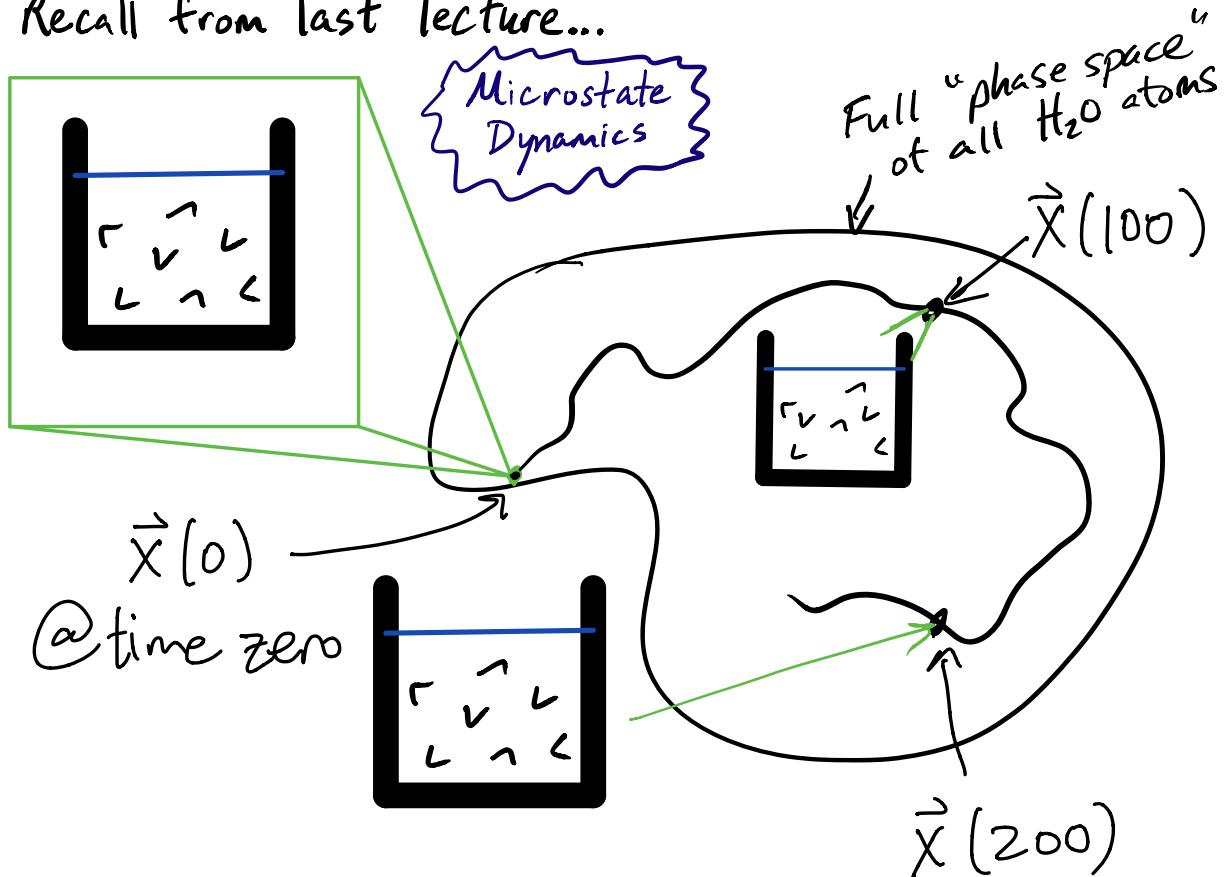


Lecture 2

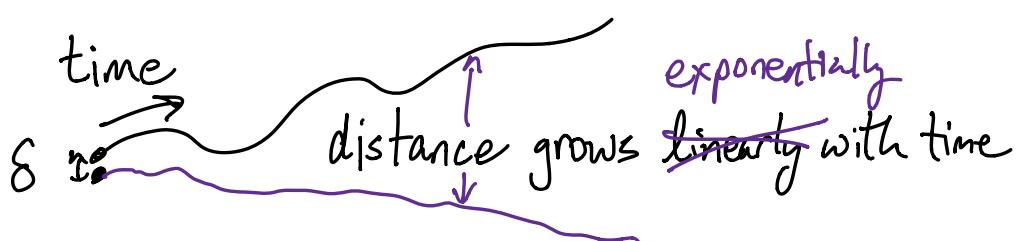
Recall from last lecture...



We discussed how it would be completely impractical to actually measure this level of detail.

Aside

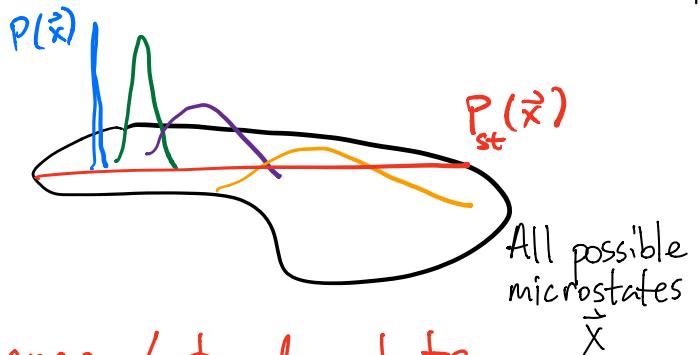
Chaos Theory



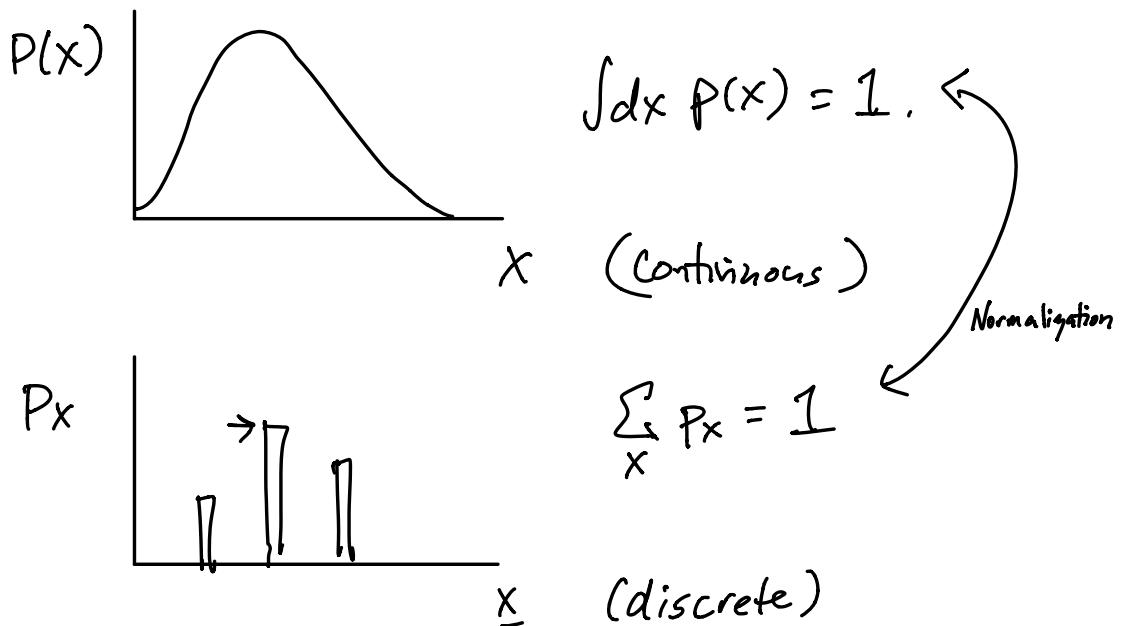
Two major types of simplifications:

A Ditch the dynamics \Rightarrow Distributions over state space

Time
0
100
200
300
 ∞

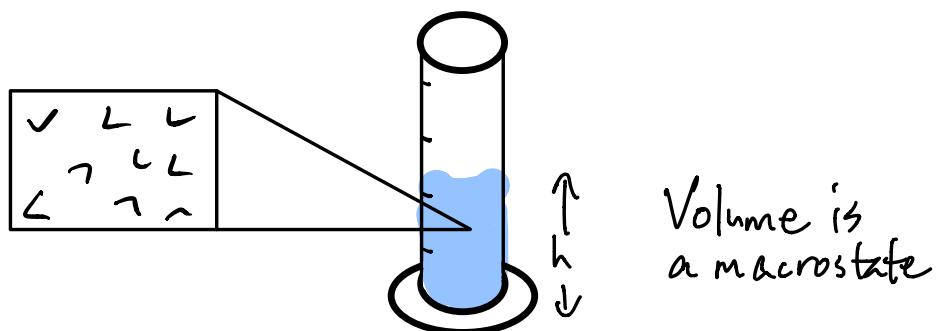


stationary / steady state



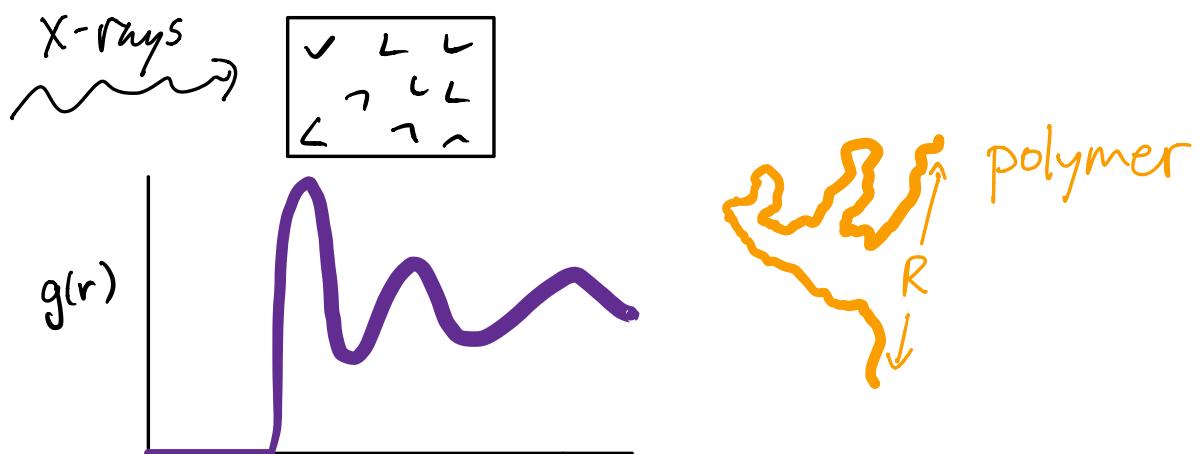
B Don't try to measure everything!
(you can't anyways)

Examples:



Volume is
a macrostate

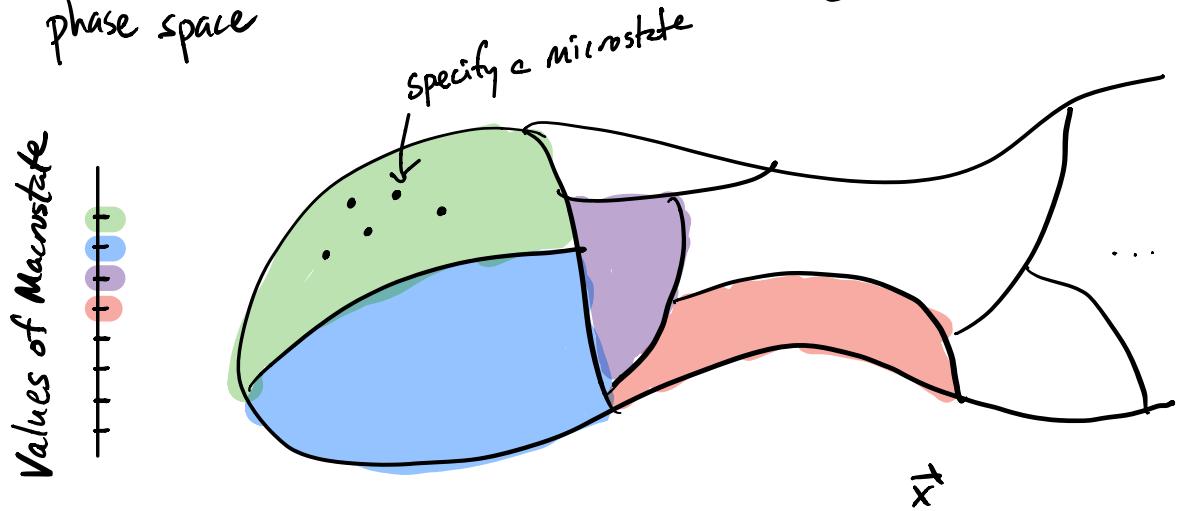
There could be many different microstates
consistent with the macroscopic measurement



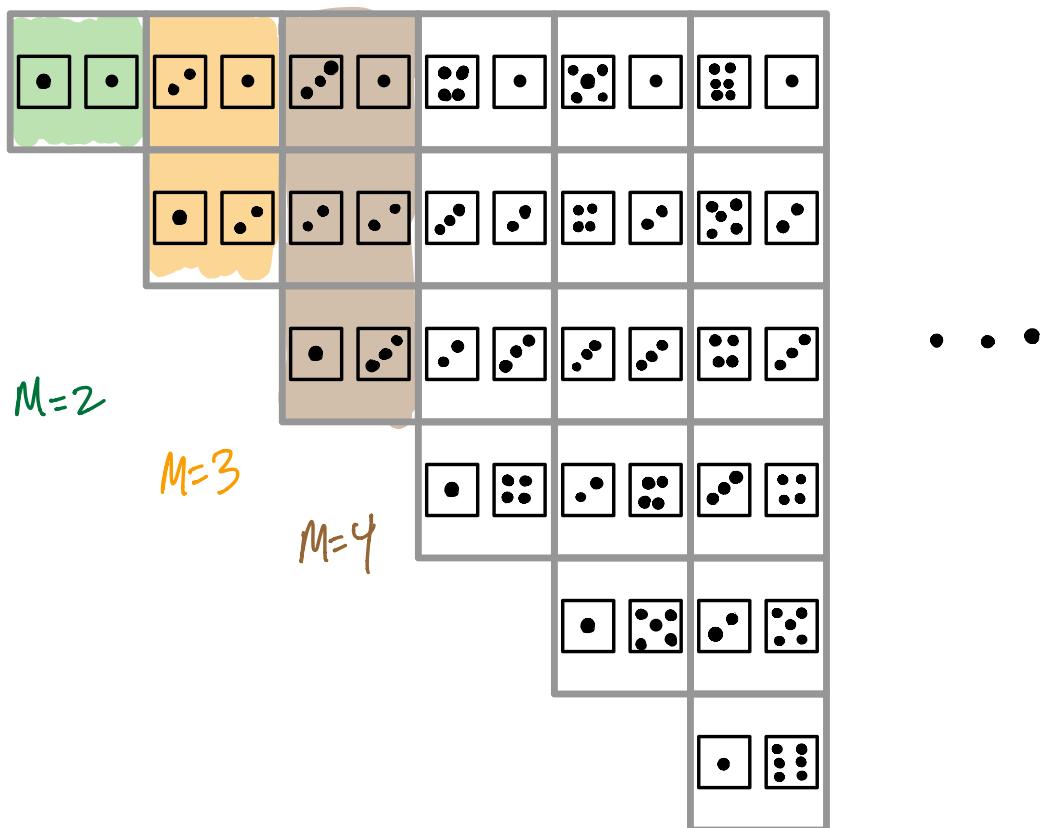
How likely you
are to find a
particle a distance
 r away from another

Macrostate is specified
by the value of R .

Macrostates correspond to a partitioning of the phase space



How probable is any one macrostate?



Every microstate is equally probable.
(Why?)

Yet every macrostate isn't

To talk quantitatively about the likelihood of macroscopic measurements, it's very important to be able to count the # of microstates consistent with a macrostate ...

$$P(M) = \sum_x P(x) \delta_{M, M(x)}$$

↑ probability of a microstate
↑ microstate

This is our many-to-one mapping

$$\delta_{i,j} : \text{Kronecker delta} = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases}$$

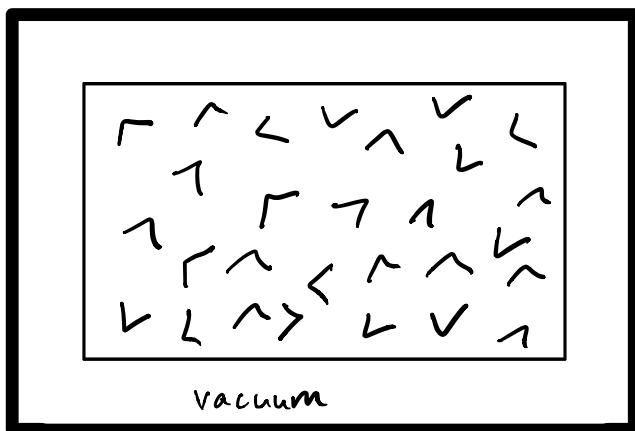
Moving from $p(x)$ → $P(M)$: Marginalization
dist. over microstates dist. over macrostates

$$P(M) = \int dx P(x) \delta(M - M(x))$$

↑
Prob. density for each microstate You only count if x corresponds to macrostate M

Let's go back to something more physical...

Closed, isolated system with N H_2O molecules

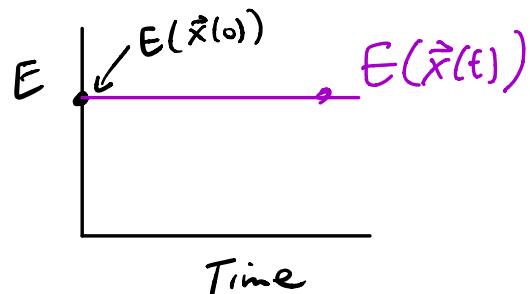
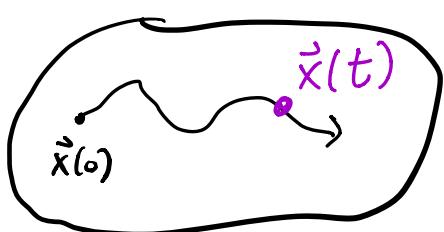


One interesting macroscopic measurement is the total energy, E .

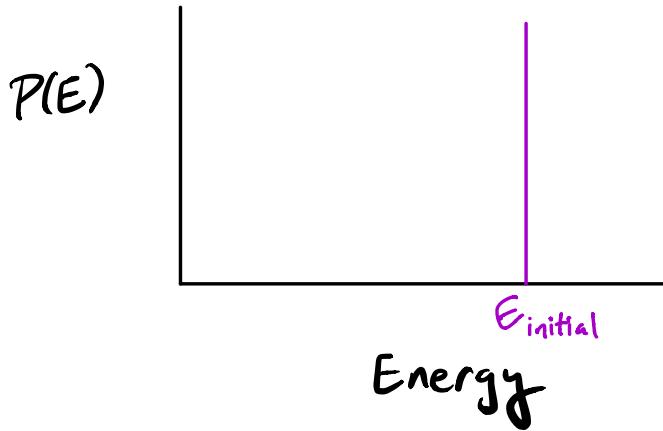
Why does a measurement of E specify a macrostate?

\exists many microstates with that same E

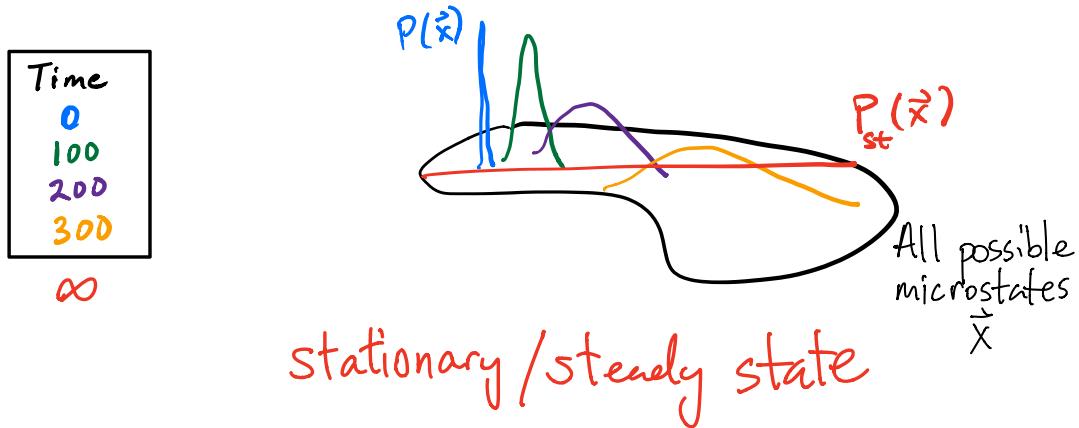
How will this macrostate change over time?



So what is $P_{st}(E)$?



What about $P(\vec{x})$?



Motivated by chaos, we guess that all microstates will ultimately be equally likely (like our dice) provided the microstate has the conserved energy E_{init} .

PRINCIPLE OF EQUAL A PRIORI WEIGHTS

$$P(\vec{x}) \propto \begin{cases} 1 & , \vec{x} \text{ satisfies conservation laws } (E = E_{\text{init}}) \\ 0 & , \text{ otherwise} \end{cases}$$

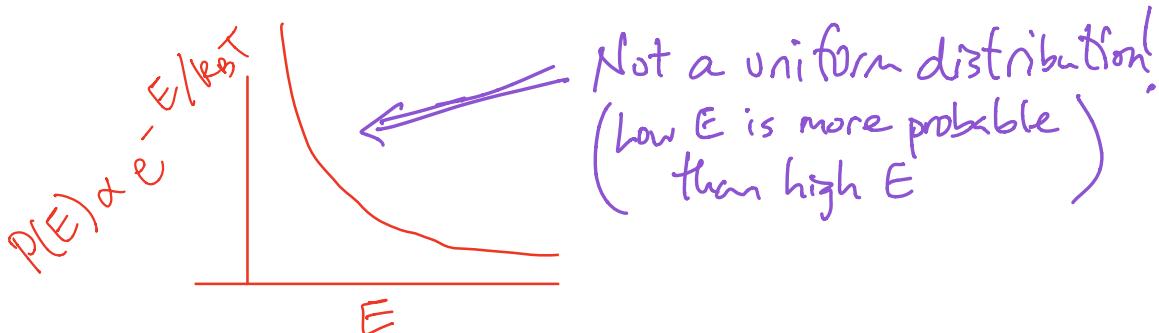
A uniform distribution - Wow! This seems crude.

Is this principle of equal a priori weights really right?

That's actually a pretty hard question to address.
See dynamical systems & ergodicity theory.

Is it useful? Most definitely

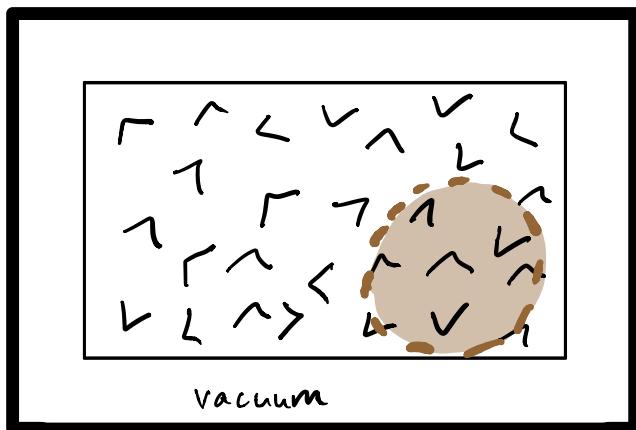
Stat Mech just assumes all possible outcomes are equally likely?! Then why did I learn about



We don't assume all distributions for all quantities are uniform! We only assumed the distribution over microstates was uniform.

Apparently making incomplete measurements can have a pretty interesting effect if we can start with a uniform distribution and end up with Boltzmann.

We are heading there next class...



What is $P(E)$ for the whole system?
... for the subsystem?